ClarityEd

Celestial Mechanics

1. A Sun-orbiting periodic comet is the farthest at 31.5 A.U. and the closest at 0.5 A.U. What is the orbital period of this comet?

2. For the comet in question 1 above, what is the area (in square A.U. per year) swept by the line joining the comet and the Sun?

3. Most single-appearance comets enter the inner Solar System directly from the Oort Cloud. Estimate how long it takes a comet to make this journey. Assume that in the Oort Cloud, 35 000 AU from the Sun the comet was at aphelion.

4. Estimate the radius of a planet that a man can escape its gravitation by jumping vertically. Assume density of the planet and the Earth are the same.

5. A spacecraft landed on the surface of a spherical asteroid with negligible rotation, whose diameter is 2.2 km, and its average density is 2.2 g/cm^3. Can the astronaut complete a circle along the equator of the asteroid on foot within 2.2 hours?

6. Estimate the mass of a globular cluster with the radius $R = 20$ pc and root mean square velocity of stars in the cluster is 3 km/s.

7. Estimate the number of stars in a globular cluster of diameter 40 pc, if the escape velocity at the edge of the cluster is 6 km/s and most of the stars are similar to the Sun.

8. On 9 March 2011 the Voyager probe was 116.406 AU from the Sun and moving at 17.062 km/s. Determine the type of orbit the probe is on:

(a) elliptical, (b) parabolic, or (c) hyperbolic.
What is the apparent magnitude of the Sun as seen from Voyager?

9. Calculate the ratio between the average densities of the Earth and the Sun, using ONLY the dataset below:
   – the angular diameter of the Sun, as seen from Earth.
   – the gravitational acceleration on Earth's surface.
   – the length of the year.
   – the fact that one degree in latitude at Earth's surface corresponds to 111 km.

10. Assuming that dust grains are black bodies, determine the diameter of a spherical dust grain which can remain at 1 AU from the Sun in equilibrium between the radiation pressure and gravitational attraction of the Sun. Take the density of the dust grain to be $\rho = 10^3$ kg m^{-3}.

11. Tidal forces result in a torque on the Earth. Assuming that, during the last several hundred million years,both this torque and the length of the sidereal year were constant and had values of6.0×10^{16} Nm and 3.15×10^7 s respectively, calculate how many days there were in a year 6.0×10^8 years ago.
    Note: Moment of inertia of a homogeneous filled sphere of radius R and mass m is
    $$I = \frac{2}{5} mR^2$$

12. A spacecraft is due to make a close pass of a space object and Scientists would like to investigate the object more carefully using a telescope on-board the spacecraft. For simplicity, let us reduce the problem to two dimensions and assume that the position of the space craft is stationary in (0,0) and the shape of the object is a disk and its boundary has the equation
    $$x^2 + y^2 - 10x - 8y + 40 = 0$$
    Find the exact values of maximum and minimum of $\tan\phi$ where ϕ is the elevation angle of the telescope with respect to the "horizontal" direction (x-axis) during investigation from one edge to the other edge.

13. Consider a Potential Hazardous Object (PHO) moving in a closed orbit under the influence of the Earth's gravitational force. Let u be the inverse of the distance of the object from the Earth and p be the magnitude of its linear momentum. As the object travels through points

A and B, values of u and p are noted as shown in the following table. Find the mass and the total energy of the object, and sketch the shape of u curve as a function of p from A to B.

	p ($\times 10^9 kgms^{-1}$)	u ($\times 10^{-8} m^{-1}$)
A	0.052	5.15
B	1.94	194.17

14. Derive a relation for the escape velocity of an object, launched from the center of a proto-star cloud. The cloud has uniform density with the mass M and radius R. Ignore collisions between the particles of the cloud and the launched object. You are given the fact that if the object were allowed to fall freely from the surface, it would reach the center with a velocity equal to $\sqrt{\frac{GM}{R}}$.

15. Gravitational forces of the Sun and the Moon lead to raising and lowering of sea water surfaces. Let ϕ be the difference in longitude between points A and B, where both points are at the equator and A is on the sea surface. Derive the horizontal acceleration of sea water at position A due to Moon's gravitational force at the time when the Moon is above point B according to observers on the Earth (express it in ϕ, the radius R of Earth, and the Earth-Moon distance r).

16. **High Altitude Projectile:** A projectile which starts from the surface of the Earth at the sea level is launched with the initial speed of $v_0 = \sqrt{\frac{GM_\oplus}{R_\oplus}}$ and with the projecting angle (with respect to the local horizon) of $\theta = \frac{\pi}{6}$. Ignore the air resistance and rotation of the Earth.
    (a) Show that the orbit of the projectile is an ellipse with a semi-major axis of $a = R_\oplus$.
    (b) Calculate the highest altitude of the projectile with respect to the Earth surface (in the unit of $R_\oplus$).
    (c) What is the range of the projectile (surface distance between launching point and falling point) in the units of $R_\oplus$?
    (d) What is eccentricity (e) of this elliptical orbit?
    (e) Find the time of flight for the projectile.

17. A spacecraft is launched from the Earth and it is quickly accelerated to its maximum velocity in the direction of the heliocentric orbit of the Earth, such that its orbit is a parabola with the Sun at its focus point, and grazes the Earth orbit. Take the orbit of the Earth and Mars as

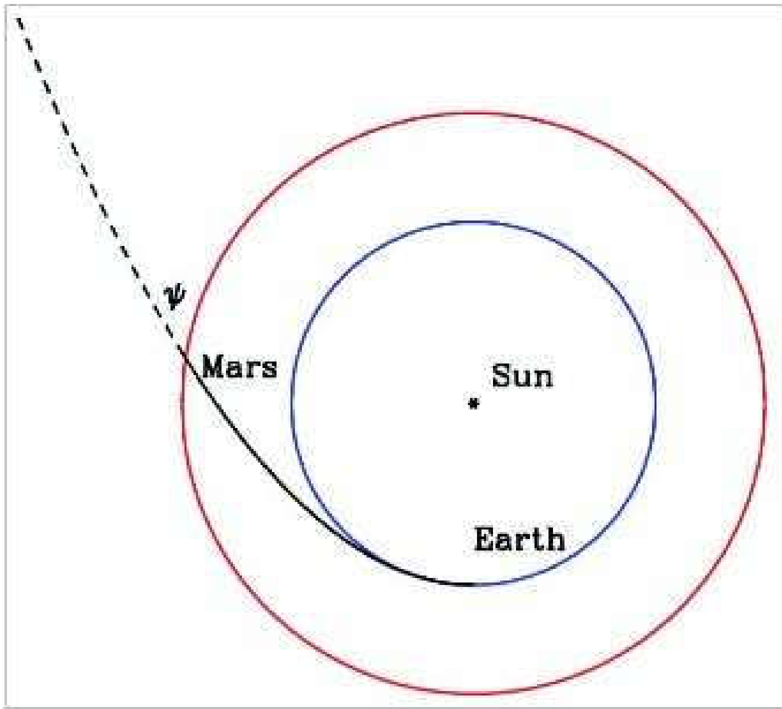

Figure 1.1 – The trajectory of the spacecraft (not in scale, seen from the north ecliptic pole). The inner circle is the orbit of the Earth, the outer circle is the orbit of Mars.

circles on the same plane. Make the following approximation: during most of the flight only the gravity from the Sun needs to be considered, but during the brief encounter with a planet, only the gravity of the planet needs to be considered.

(a) What is the angle (ψ) between the path of the spacecraft and the orbit of the Mars (see Figure 1.1) as it crosses the orbit of the Mars, without considering the gravity effect of the Mars?

(b) Suppose the Mars happens to be very close to the crossing point at the time of the crossing, from the point of view of an observer on Mars, what is the approaching velocity and direction of approach (with respect to the Sun) of the spacecraft before it is significantly affected by the gravity of the Mars?

18. A satellite orbits the Earth on a circular orbit. The initial momentum of the satellite is given by the vector $\overrightarrow{p}$. At a certain time, an explosive charge is set off which gives the satellite an additional impulse $\Delta\overrightarrow{p}$, equal in magnitude to $|\overrightarrow{p}|$.

Let α be the angle between the vectors $\overrightarrow{p}$ and $\Delta\overrightarrow{p}$,and β between the radius vector of the satellite and the vector $\Delta\overrightarrow{p}$.

By thinking about the direction of the additional impulse $\Delta\overrightarrow{p}$, consider if it is possible to change the orbit to each of the cases given below. If it is possible and give values of α and β for which it is possible. If the orbit is not possible, mark NO.

(a) a hyperbola with perigee at the location of the explosion.

(b) a parabola with perigee at the location of the explosion.

(c) an ellipse with perigee at the location of the explosion.

(d) a circle.

(e) an ellipse with apogee at the location of the explosion.

Note that for $\alpha = 180°$ and $\beta = 90°$ the new orbit will be a line along which the satellite will free fall vertically towards the centre of the Earth.