4. The roots of the equation $x^2 - (m-3)x + m = 0$ are such that exactly one of them lies in the interval $(1, 2)$. Then

(A) $5 < m < 7$ (B) $m < 10$
(C) $2 < m < 5$ (D) $m > 10$

5. If α and β are roots of the equation $2x^2 + ax + b = 0$, then one of the roots of the equation $2(\alpha x + \beta)^2 + a(\alpha x + \beta) + b = 0$ is

(A) 0 (B) $\dfrac{\alpha + 2b}{\alpha^2}$
(C) $\dfrac{a\alpha + b}{2\alpha^2}$ (D) $\dfrac{a\alpha - 2b}{2\alpha^2}$

6. If $a < b$ and $x^2 + (a+b)x + ab < 0$, then

(A) $a < x < b$ (B) $-b < x < -a$
(C) $x < a$ or $x > b$ (D) $x < -b$ or $x > -a$

7. If α and β are the roots of $x^2 - 2x + 4 = 0$, then the value of $\alpha^6 + \beta^6$ is

(A) 64 (B) 128 (C) 256 (D) 32

8. The greatest value of the expression

$$\frac{1}{4t^2 + 2t + 1}$$

is

(A) 4/3 (B) 5/2 (C) 13/14 (D) 14/13

9. The roots of the equation $4^x - 3 \times 2^{x+2} + 32 = 0$ are

(A) 1, 2 (B) 1, 3 (C) 2, 3 (D) 2, 1/2

10. If the equations $x^2 - 3x + a = 0$ and $x^2 + ax - 3 = 0$ have a common root, then a possible value of a is

(A) 3 (B) 1 (C) –2 (D) 2

11. If $x^2 - 1 \le 0$ and $x^2 - x - 2 \ge 0$ hold simultaneously for a real x, then x belongs to the interval

(A) $(-1, 2)$ (B) $(-1, 1)$
(C) $[-1, 2)$ (D) $x = -1$

12. Let $\alpha \ne 1$ and $\alpha^{13} = 1$. If $a = \alpha + \alpha^3 + \alpha^4 + \alpha^{-4} + \alpha^{-3} + \alpha^{-1}$ and $b = \alpha^2 + \alpha^5 + \alpha^6 + \alpha^{-6} + \alpha^{-5} + \alpha^{-2}$ then the quadratic equation whose roots are a and b is

(A) $x^2 + x + 3 = 0$ (B) $x^2 + x + 4 = 0$
(C) $x^2 + x - 3 = 0$ (D) $x^2 + x - 4 = 0$

13. If $ax^2 - 2a^2x + 1 = 0$ and $x^2 - 3ax + a^2 = 0$, $a \ne 0$, have a common root, then a^3 is a root of the equation

(A) $x^2 - x - 1 = 0$ (B) $x^2 + x - 1 = 0$
(C) $x^2 + x + 1 = 0$ (D) $x^2 - x - 2 = 0$

14. A sufficient condition for the equation $x^2 + bx - 4 = 0$ to have integer roots is that

(A) $b = 0, \pm 3$ (B) $b = 0, \pm 2$
(C) $b = 0, \pm 1$ (D) $b = 0, \pm 4$

15. The quadratic expression $ax^2 + bx + c$ assumes both positive and negative values if and only if

(A) $ab \ne 0$ (B) $b^2 - 4ac > 0$
(C) $b^2 - 4ac \ge 0$ (D) $b^2 - 4ac < 0$

16. If $a > 0$ and one root of $ax^2 + bx + c = 0$ is less than –2 and the other is greater than 2, then

(A) $4a + 2|b| + c < 0$
(B) $4a + 2|b| + c > 0$
(C) $4a + 2|b| + c = 0$
(D) $a + b = c$

17. If b and c are real, then the equation $x^2 + bx + c = 0$ has both roots real and positive if and only if

(A) $b < 0$ and $c > 0$
(B) $bc < 0$ and $b \ge 2\sqrt{c}$
(C) $bc < 0$ and $b^2 \ge 4c$
(D) $c > 0$ and $b \le -2\sqrt{c}$

18. It is given that the quadratic expression $ax^2 + bx + c$ takes all negative values for all x less than 7. Then

(A) $ax^2 + bx + c = 0$ has equal roots
(B) a is negative
(C) a and b are both negative
(D) a and b are both positive

19. The value of a for which the equation $\cos^4 x - (a+2)\cos^2 x - (a+3) = 0$ possesses solution, belongs to the interval

(A) $(-\infty, 3)$ (B) $(2, +\infty)$
(C) $[-3, -2]$ (D) $(0, +\infty)$

20. If the expression $ax + (1/x) - 2 \ge 0$ for all positive values of x, then the minimum value of a is

(A) 1 (B) 2 (C) 1/4 (D) 1/2

21. If a, b and c are real, $a \ne 0, b \ne c$ and the equations $x^2 + abx + c = 0$ and $x^2 + cax + b = 0$ have a common root, then

(A) $a^2(b + c) = -1$ (B) $b^2(c + a) = 1$
(C) $c^2(a + b) = 1$ (D) $a^2(b + c) = 1$

22. If $(x^2 + x + 2)^2 - (a-3)(x^2 + x + 2)(x^2 + x + 1) + (a-4) \times (x^2 + x + 1)^2 = 0$ has atleast one real root, then

(A) $0 < a < 5$ (B) $5 < a \le 19/3$
(C) $5 \le a < 7$ (D) $a \ge 7$

1. The equation $x^{(3/4)(\log_2 x)^2 + \log_2 x - (5/4)} = \sqrt{2}$ has

(A) atleast one real solution
(B) exactly three solutions
(C) exactly one irrational solution
(D) complex roots

2. If S is the set of all real values of x such that

$$\frac{2x-1}{2x^3+3x^2+x} > 0$$

then S is a superset of

(A) $(-\infty, -3/2)$ (B) $(-3/2, -1/4)$
(C) $(-1/4, 1/2)$ (D) $(1/2, 3)$

3. If $||x^2-5x+4|-|2x-3|| = |x^2-3x+1|$, then x belongs to the interval

(A) $(-\infty, 1]$ (B) $(1, 3/2)$
(C) $[3/2, 4]$ (D) $(4, +\infty)$

4. Let

$$y = \sqrt{\frac{(x+1)(x-3)}{x-2}}$$

Then the set of real values of x for which y is real is

(A) $[-1, 2)$ (B) $(2, 3)$
(C) $(-\infty, -1)$ (D) $[3, +\infty)$

5. Let a, b and c be distinct positive reals such that the quadratics ax^2+bx+c, bx^2+cx+a and cx^2+ax+b are all positive for all real x and

$$s = \frac{a^2+b^2+c^2}{ab+bc+ca}$$

Then

(A) $s \le 1$ (B) $1 < s < 4$
(C) $s \notin (-\infty, 1) \cup (4, +\infty)$ (D) $0 < s < 1$

6. If α and $1/\alpha$ $(\alpha > 0)$ are roots of $ax^2 - bx + c = 0$, then

(A) $c = a$ (B) $c \ge 2b$
(C) $b \ge 2a$ (D) $a \ge 2b$

7. If

$$\frac{k}{2x} = \frac{a}{x+c} + \frac{b}{x-c}$$

where $c \ne 0$, a and b are positive, has equal roots, then the value of k is

(A) $a+b$ (B) $a-b$
(C) $(\sqrt{a}+\sqrt{b})^2$ (D) $(\sqrt{a}-\sqrt{b})^2$

8. If the product of the roots of the equation

$$x^2 - 4mx + 3e^{2\log m} - 4 = 0$$

is 8, then the roots are

(A) real (B) non-real
(C) rational (D) irrational

9. If $3^{-\log_{1/9}[x^2-(10/3)x+1]} \le 1$, then x belongs to

(A) $[0, 1/3)$ (B) $(1/3, 1)$
(C) $(2, 3)$ (D) $(3, 10/3]$

10. If every pair of the equations $x^2+ax+bc=0$, $x^2+bx+ca=0$ and $x^2+cx+ab=0$ has a common root, then

(A) sum of these common roots is $-(1/2)(a+b+c)$
(B) sum of these common roots is $(1/2)(a+b+c)$
(C) product of the common roots is abc
(D) product of the common roots is $-(abc)$

11. If the equations $4x^2-11x+2k=0$ and $x^2-3x-k=0$ have a common root α, then

(A) $k = 0$ (B) $k = -17/36$
(C) $\alpha = 0$ (D) $\alpha = 17/6$

12. If a and b are real and $x^2+ax+b^2=0$ and $x^2+ax+a^2=0$ have a common root, then which of the following are true?

(A) $a = b$
(B) $a+b$ is the common root
(C) for real roots, $a = b = 0$
(D) no real values of a and b exist

13. For $a > 1$, the equation

$$(a+\sqrt{a^2-1})^{x^2-2x} + (a-\sqrt{a^2-1})^{x^2-2x} = 2a$$

has

(A) three real roots
(B) roots which are independent of a
(C) roots whose sum is 3
(D) roots whose product is -1

14. If a, b and c are positive real and $a = 2b+3c$, then the equation $ax^2+bx+c=0$ has real roots for

(A) $\left|\frac{b}{c}-4\right| \ge 2\sqrt{7}$ (B) $\left|\frac{c}{b}-4\right| \ge 2\sqrt{7}$
(C) $\left|\frac{a}{c}-11\right| \ge 4\sqrt{7}$ (D) $\left|\frac{a}{b}+4\right| \ge 2\sqrt{\frac{13}{3}}$